**Printed Page:-04** **Subject Code:- AMIAS0203**

**Roll. No:**

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**NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA**

**(An Autonomous Institute Affiliated to AKTU, Lucknow)**

**M.Tech (Integrated)**

**SEM: II - THEORY EXAMINATION (2023- 2024)**

**Subject: Engineering Mathematics-II**

**Time: 3 Hours** **Max. Marks: 100**

**General Instructions:**

**IMP:** *Verify that you have received the question paper with the correct course, code, branch etc.*

***1.*** *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

***2.*** *Maximum marks for each question are indicated on right -hand side of each question.*

***3.*** *Illustrate your answers with neat sketches wherever necessary.*

***4.*** *Assume suitable data if necessary.*

***5.*** *Preferably, write the answers in sequential order.*

***6.*** *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

**SECTION-A** 20

1. Attempt all parts:-

1-a. Degree and order of the differential equation $\sqrt{((dy/dx)^2 + 3y)} = (d^2y)/(dx^2)$ is (CO1) 1

(a) Ord = 2, Deg = 2

(b) Ord = 2, Deg = 1

(c) Ord = 1, Deg = 2

(d) Ord = 1, Deg = 1

1-b. The P.I. of the differential equation $(D^2 + 4)y = \sin 3x$ is (CO1) 1

(a) $(-1/10)\sin 3x$

(b) $(-1/5)\sin 3x$

(c) $(1/5)\sin 3x$

(d) None of these

1-c. The coefficient $b_n$ in a Fourier series for the function $f(x) = x\sin x$ in the interval $-\pi < x < \pi$ is (CO2) 1

(a) $\pi$

(b) 0

(c) 1

(d) $2\pi$

1-d. For the series $\sum_{n=1}^{\infty} u_n$ of positive terms the Ratio test fails if $\lim_{n\to\infty} \frac{u_n}{u_{n+1}}$ is (CO2) 1

(a) <1
(b) >1
(c) = 1
(d) none of these

1-e. Laplace transform of the function $F(t) = t^n$, where n is a positive integer is (CO3) 1

(a) $\frac{n}{s^n+1}$
(b) $\frac{1}{s^n+1}$
(c) $\frac{1}{s^{n+1}}$
(d) None of these

1-f. Inverse Laplace of the function $f(s) = \frac{1}{(s+1)^2}$ is (CO3) 1

(a) $te^t$
(b) $\frac{e^{-t}}{t}$
(c) $te^{-t}$
(d) None of these

1-g. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $div\vec{r}$ equal to (CO4) 1

(a) 4
(b) 8
(c) 5
(d) 3

1-h. If $\emptyset = 3x^2y - y^3z^2$ then grad $\emptyset$ at the point $(1,-2,-1)$ is (CO4) 1

(a) $-12\hat{i} - 9\hat{j} - 16\hat{k}$
(b) $12\hat{i} + 9\hat{j} + 16\hat{k}$
(c) $\hat{i} + \hat{j} + \hat{k}$
(d) $\hat{i} + \hat{j}$

1-i. A boy has coins in the denominations of ₹ 1 and ₹ 2. If he has total 30 coins and the value of coins is ₹ 48. Find the number of ₹ 1 coins he has. (CO5) 1

(a) 18
(b) 10
(c) 12

(d) 14

1-j. Introducing a boy, a girl said, "He is the son of the daughter of the father of my uncle." How is the boy related to the girl? (CO5) 1

(a) Brother
(b) Nephew
(c) Uncle
(d) Son-in-law

2. Attempt all parts:-

2.a. Solve the differential equation: $(D^3 - 3D^2 + 4)y = 0.$ (CO1) 2

2.b. **Find Fourier coefficients $a_0$, in $(0, 2)$ for** $f(x) = \begin{pmatrix} x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{pmatrix}$ (CO2) 2

2.c. Find Laplace transform of the function $F(t) = \cos h\ at - \cos at.$ (CO3) 2

2.d. Prove that $\vec{V} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$ is solenoidal. (CO4) 2

2.e. The ratio between the ages of Kamala and Savitri is 6:5 and the sum of their ages is 44 years. Find the ratio of their ages after 8 years? (CO5) 2

**SECTION-B** 30

3. Answer any five of the following:-

3-a. Solve the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin(\log x^2).$ (CO1) 6

3-b. Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \sin x.$ (CO1) 6

3-c. Show that the series $1 + \frac{1}{2^2} + \frac{2^2}{3^2} + \frac{3^2}{4^2} + \ldots\ldots\ldots$ is divergent. (CO2) 6

3-d. Expand f(x) = π x - $x^2$ as a Fourier half range sine series in 0 < x < π upto the first three terms. (CO2) 6

3.e. Find the inverse Laplace Transform of the function $f(s) = \frac{s^2 + 2s - 3}{s(s-3)(s+2)}$. (CO3) 6

3.f. Find the total work done by a force $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ in moving a point from (0,0) to (a,b) along the rectangle bounded by the lines x=0, x=a, y=0, and y=b. (CO4) 6

3.g. (i) 12 yr ago the ratio between the ages of A and B was 3:4 respectively. The present age of A is 3 times of C's present age. If C's present age is 10 yr, then what is B's present age? 6

(ii) The ratio of the present ages of a mother and daughter is 7:1 Four years ago the ratio of their ages was 19:1 what will be the mother's age four years from now ? (CO5)

**SECTION-C** 50

4. Answer any one of the following:-

4-a. Solve the following differential equation by changing to independent variable 10

$x\frac{d^2y}{dx^2}+(4x^2-1)\frac{dy}{dx}+4x^3y=2x^3.$ (CO1)

4-b. 10

Solve $\frac{dx}{dt}+2x-3y=t,\ \frac{dy}{dt}-3x+2y=e^{2t}.$ (CO1)

5. Answer any one of the following:-

5-a. Obtain the Fourier series to represent function $f(x)=4-x^2$ in the interval $-2\le x\le 2$. 10

Hence show that $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots\ldots=\frac{\pi^2}{12}$

5-b. $f(x)=x\cos x$ 10

Obtain the Fourier Series to represent the function in the interval $-\pi<x<\pi$. (CO2)

6. Answer any one of the following:-

6-a. Solve the following differential equation by using Laplace transform 10

$\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t,\ \ Given\ that\ x=2,\ \frac{dx}{dt}=-1\ at\ t=0.$ (CO3)

6-b. Using Convolution Theorem find the inverse Laplace transform of 10

$L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$. (CO3)

7. Answer any one of the following:-

7-a. 10

Evaluate $\oint\vec{F}.d\vec{r}$, by stokes theorem where $\vec{F}=y^2\hat{i}+x^2\vec{j}-(x+z)\hat{k}$ and C is the boundary of the triangle with vertices at $(0,0,0),(1,0,0)$ and $(1,1,0)$. (CO4)

7-b. 10

Verify Divergence theorem for $\vec{F}=4xz\,\hat{i}-y^2\hat{j}+yz\,\hat{k}$ taken over the cube bounded by the planes
x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. (CO4)

8. Answer any one of the following:-

8-a. (i) Two numbers are in the ratio 3: 5. If 9 be subtracted from each, then they are in the ratio of 12:23. Find the second number ? 10

(ii) Alloy A contains 40% gold and 60% silver. Alloy B contains 35% gold and 40% silver and 25% copper. Alloy A and B are mixed in the ratio of 1:4. What is the ratio of gold and silver in the newly formed alloy? (CO5)

8-b. (i) Vinod starts from his house and travels 4 km in East direction after that he turns towards left and moves 4 km. Finally, he turns towards left and moves 4 km. At what distance and in which direction he finally stands from his starting point? 10

(ii) A person moves 15 km in East direction then turns towards North and moves 4 km. From here he turns towards West and travels 12 km. How far and in which direction is he from his starting point? (CO5)